

## STEP III, 2010 Q10

- 10 A small bead  $B$ , of mass  $m$ , slides without friction on a fixed horizontal ring of radius  $a$ . The centre of the ring is at  $O$ . The bead is attached by a light elastic string to a fixed point  $P$  in the plane of the ring such that  $OP = b$ , where  $b > a$ . The natural length of the elastic string is  $c$ , where  $c < b - a$ , and its modulus of elasticity is  $\lambda$ . Show that the equation of motion of the bead is

$$ma\ddot{\phi} = -\lambda \left( \frac{a \sin \phi}{c \sin \theta} - 1 \right) \sin(\theta + \phi),$$

where  $\theta = \angle BPO$  and  $\phi = \angle BOP$ .

Given that  $\theta$  and  $\phi$  are small, show that  $a(\theta + \phi) \approx b\theta$ . Hence find the period of small oscillations about the equilibrium position  $\theta = \phi = 0$ .



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