

STEP III, 2010 Q1

- 1 Let x_1, x_2, \dots, x_n and x_{n+1} be any fixed real numbers. The numbers A and B are defined by

$$A = \frac{1}{n} \sum_{k=1}^n x_k, \quad B = \frac{1}{n} \sum_{k=1}^n (x_k - A)^2,$$

and the numbers C and D are defined by

$$C = \frac{1}{n+1} \sum_{k=1}^{n+1} x_k, \quad D = \frac{1}{n+1} \sum_{k=1}^{n+1} (x_k - C)^2.$$

- (i) Express C in terms of A , x_{n+1} and n .

- (ii) Show that $B = \frac{1}{n} \sum_{k=1}^n x_k^2 - A^2$.

- (iii) Express D in terms of B , A , x_{n+1} and n .

Hence show that $(n+1)D \geq nB$ for all values of x_{n+1} , but that $D < B$ if and only if

$$A - \sqrt{\frac{(n+1)B}{n}} < x_{n+1} < A + \sqrt{\frac{(n+1)B}{n}}.$$



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