

STEP III, 2009, Q8

8 Let m be a positive integer and let n be a non-negative integer.

(i) Use the result $\lim_{t \rightarrow \infty} e^{-mt}t^n = 0$ to show that

$$\lim_{x \rightarrow 0} x^m(\ln x)^n = 0.$$

By writing x^x as $e^{x \ln x}$ show that

$$\lim_{x \rightarrow 0} x^x = 1.$$

(ii) Let $I_n = \int_0^1 x^m(\ln x)^n dx$. Show that

$$I_{n+1} = -\frac{n+1}{m+1}I_n$$

and hence evaluate I_n .

(iii) Show that

$$\int_0^1 x^x dx = 1 - \left(\frac{1}{2}\right)^2 + \left(\frac{1}{3}\right)^3 - \left(\frac{1}{4}\right)^4 + \dots.$$



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