

STEP III, 2009, Q7

- 7 (i) The functions $f_n(x)$ are defined for $n = 0, 1, 2, \dots$, by

$$f_0(x) = \frac{1}{1+x^2} \quad \text{and} \quad f_{n+1}(x) = \frac{df_n(x)}{dx}.$$

Prove, for $n \geq 1$, that

$$(1+x^2)f_{n+1}(x) + 2(n+1)xf_n(x) + n(n+1)f_{n-1}(x) = 0.$$

- (ii) The functions $P_n(x)$ are defined for $n = 0, 1, 2, \dots$, by

$$P_n(x) = (1+x^2)^{n+1}f_n(x).$$

Find expressions for $P_0(x)$, $P_1(x)$ and $P_2(x)$.

Prove, for $n \geq 0$, that

$$P_{n+1}(x) - (1+x^2)\frac{dP_n(x)}{dx} + 2(n+1)xP_n(x) = 0,$$

and that $P_n(x)$ is a polynomial of degree n .



NextStepMaths.com

To view mark schemes, fully worked solutions and examiner's comments, and for more details about tutoring and other services offered, go to

NextStepMaths.com