

STEP III, 2009, Q5

- 5 The numbers x , y and z satisfy

$$\begin{aligned}x + y + z &= 1 \\x^2 + y^2 + z^2 &= 2 \\x^3 + y^3 + z^3 &= 3.\end{aligned}$$

Show that

$$yz + zx + xy = -\frac{1}{2}.$$

Show also that $x^2y + x^2z + y^2z + y^2x + z^2x + z^2y = -1$, and hence that

$$xyz = \frac{1}{6}.$$

Let $S_n = x^n + y^n + z^n$. Use the above results to find numbers a , b and c such that the relation

$$S_{n+1} = aS_n + bS_{n-1} + cS_{n-2},$$

holds for all n .



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