

STEP III, 2009 Q4 MS

4. (i) Substituting into the definition yields the Laplace transform as

$$\int_0^{\infty} e^{-st} e^{-bt} f(t) dt = \int_0^{\infty} e^{-t(s+b)} f(t) dt = F(s+b)$$

(ii) Similarly, a change of variable in the integral using $u = at$ yields the result.

(iii) Integrating by parts yields this answer.

(iv) A repeated integration by parts obtains

$$F(s) = 1 - s^2 F(s)$$

which leads to the stated result.

Using the results obtained in the question, the transform of $\cos qt$ is

$$q^{-1} \left(\frac{s/q}{s^2/q^2 + 1} \right) = \frac{s}{s^2 + q^2}, \text{ and so the transform of } e^{-pt} \cos qt \text{ is } \frac{(s+p)}{(s+p)^2 + q^2}$$



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