# STEP III, 2009 Q4 MS

4. (i) Substituting into the definition yields the Laplace transform as

$$\int_0^\infty e^{-st}e^{-bt}f(t)dt = \int_0^\infty e^{-t(s+b)}f(t)dt = F(s+b)$$

(ii) Similarly, a change of variable in the integral using u = at yields the result.

(iii) Integrating by parts yields this answer.

(iv) A repeated integration by parts obtains

$$F(s) = 1 - s^2F(s)$$

which leads to the stated result.

Using the results obtained in the question, the transform of $\cos qt$ is

$q^{-1}\left(\frac{s/q}{s^2/q^2+1}\right) = \frac{s}{s^2+q^2}$ ,and so the transform of $e^{-pt}\cos qt$ is $\frac{(s+p)}{(s+p)^2+q^2}$