

STEP III, 2009, Q4

- 4 For any given (suitable) function f , the *Laplace transform* of f is the function F defined by

$$F(s) = \int_0^{\infty} e^{-st}f(t)dt \quad (s > 0).$$

- (i) Show that the Laplace transform of $e^{-bt}f(t)$, where $b > 0$, is $F(s + b)$.
- (ii) Show that the Laplace transform of $f(at)$, where $a > 0$, is $a^{-1}F(\frac{s}{a})$.
- (iii) Show that the Laplace transform of $f'(t)$ is $sF(s) - f(0)$.
- (iv) In the case $f(t) = \sin t$, show that $F(s) = \frac{1}{s^2 + 1}$.

Using only these four results, find the Laplace transform of $e^{-pt} \cos qt$, where $p > 0$ and $q > 0$.



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