

STEP III, 2009, Q3

3 The function $f(t)$ is defined, for $t \neq 0$, by

$$f(t) = \frac{t}{e^t - 1}.$$

- (i) By expanding e^t , show that $\lim_{t \rightarrow 0} f(t) = 1$. Find $f'(t)$ and evaluate $\lim_{t \rightarrow 0} f'(t)$.
- (ii) Show that $f(t) + \frac{1}{2}t$ is an even function. [Note: A function $g(t)$ is said to be *even* if $g(t) \equiv g(-t)$.]
- (iii) Show with the aid of a sketch that $e^t(1 - t) \leq 1$ and deduce that $f'(t) \neq 0$ for $t \neq 0$.

Sketch the graph of $f(t)$.



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