STEP III, 2009 Q2 MS

2 (i) The five required results are straightforward to write down, merely observing that initial terms in the summations are zero.

(ii) Substituting the series from (i) in the differential equation yields that $-a_1 + 3a_3x^2 + (8a_4 + 4a_0)x^3 + ... = 0$, after having collected like terms.
Thus, comparing constants and x^2 coefficients $a_1 = 0$ and $a_3 = 0$
Comparing coefficients of x^{n-1}, for $n \geq 4$, $n(n-1)a_n - na_n + 4a_{n-4} = 0$ which gives the required result upon rearrangement.

With $a_0 = 1$, $a_2 = 0$, and as $a_1 = 0$, and $a_3 = 0$, we find $a_4 = \frac{-1}{2!}$, $a_5 = 0$, $a_6 = 0$, $a_7 = 0$, $a_8 = \frac{1}{4!}$, etc.

Thus $y = 1 - \frac{1}{2!}(x^2)^2 + \frac{1}{4!}(x^2)^4 - \frac{1}{6!}(x^2)^6 + ... = \cos(x^2)$

With $a_0 = 0$, $a_2 = 1$, $y = (x^2) - \frac{1}{3!}(x^2)^3 + \frac{1}{5!}(x^2)^5 - \frac{1}{7!}(x^2)^7 + ... = \sin(x^2)$