

## STEP III, 2009, Q2

- 2 (i) Let  $y = \sum_{n=0}^{\infty} a_n x^n$ , where the coefficients  $a_n$  are independent of  $x$  and are such that this series and all others in this question converge. Show that

$$y' = \sum_{n=1}^{\infty} n a_n x^{n-1},$$

and write down a similar expression for  $y''$ .

Write out explicitly each of the three series as far as the term containing  $a_3$ .

- (ii) It is given that  $y$  satisfies the differential equation

$$x y'' - y' + 4x^3 y = 0.$$

By substituting the series of part (i) into the differential equation and comparing coefficients, show that  $a_1 = 0$ .

Show that, for  $n \geq 4$ ,

$$a_n = -\frac{4}{n(n-2)} a_{n-4},$$

and that, if  $a_0 = 1$  and  $a_2 = 0$ , then  $y = \cos(x^2)$ .

Find the corresponding result when  $a_0 = 0$  and  $a_2 = 1$ .



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