

STEP III, 2009 Q1 MS

1. The result for p can be found via calculating the equation of the line SV

($y - ms = \frac{ms - nv}{s - v}(x - s)$) or similar triangles. The result for q follows from that for p (given in the question) by suitable interchange of letters to give

$$q = \frac{(m - n)tu}{mt - nu}$$

As S and T lie on the circle, s and t are solutions of the equation

$$\lambda^2 + (m\lambda - c)^2 = r^2 \quad \text{i.e.} \quad (1 + m^2)\lambda^2 - 2mc\lambda + (c^2 - r^2) = 0$$

and so from considering sum and product of roots, $st = \frac{c^2 - r^2}{1 + m^2}$, and $s + t = \frac{2mc}{1 + m^2}$

Similarly $uv = \frac{c^2 - r^2}{1 + n^2}$, and $u + v = \frac{2nc}{1 + n^2}$ can be deduced by interchanging letters.

Substituting from the earlier results $p + q = \frac{(m - n)sv}{ms - nv} + \frac{(m - n)tu}{mt - nu}$ which can

be simplified to $\frac{(m - n)}{(ms - nv)(mt - nu)}(stm(u + v) - nuv(s + t))$

and then substituting the sum and product results yields the required result.



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