

STEP III, 2009, Q13

- 13 (i) The point P lies on the circumference of a circle of unit radius and centre O . The angle, θ , between OP and the positive x -axis is a random variable, uniformly distributed on the interval $0 \leq \theta < 2\pi$. The cartesian coordinates of P with respect to O are (X, Y) . Find the probability density function for X , and calculate $\text{Var}(X)$.

Show that X and Y are uncorrelated and discuss briefly whether they are independent.

- (ii) The points P_i ($i = 1, 2, \dots, n$) are chosen independently on the circumference of the circle, as in part (i), and have cartesian coordinates (X_i, Y_i) . The point \bar{P} has coordinates (\bar{X}, \bar{Y}) , where $\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i$ and $\bar{Y} = \frac{1}{n} \sum_{i=1}^n Y_i$. Show that \bar{X} and \bar{Y} are uncorrelated.

Show that, for large n , $P\left(|\bar{X}| \leq \sqrt{\frac{2}{n}}\right) \approx 0.95$.



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