

## STEP III, 2009, Q12

- 12 (i) Albert tosses a fair coin  $k$  times, where  $k$  is a given positive integer. The number of heads he gets is  $X_1$ . He then tosses the coin  $X_1$  times, getting  $X_2$  heads. He then tosses the coin  $X_2$  times, getting  $X_3$  heads. The random variables  $X_4, X_5, \dots$  are defined similarly. Write down  $E(X_1)$ .

By considering  $E(X_2 \mid X_1 = x_1)$ , or otherwise, show that  $E(X_2) = \frac{1}{4}k$ .

Find  $\sum_{i=1}^{\infty} E(X_i)$ .

- (ii) Bertha has  $k$  fair coins. She tosses the first coin until she gets a tail. The number of heads she gets before the first tail is  $Y_1$ . She then tosses the second coin until she gets a tail and the number of heads she gets with this coin before the first tail is  $Y_2$ . The random variables  $Y_3, Y_4, \dots, Y_k$  are defined similarly, and  $Y = \sum_{i=1}^k Y_i$ .

Obtain the probability generating function of  $Y$ , and use it to find  $E(Y)$ ,  $\text{Var}(Y)$  and  $P(Y = r)$ .



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