

## STEP III, 2009 Q11 MS

11. (i) Conserving momentum yields  $MV = M(1 + bx)v$  and so  $V = (1 + bx)v$ . Written as  $V = (1 + bx)\frac{dx}{dt}$ , separating variables and integrating  $Vt + c = x + \frac{1}{2}bx^2$ , but as  $= 0$ , when  $t = 0$ ,  $c = 0$   
 So  $\frac{1}{2}bx^2 + x - Vt = 0$ , and so  $x = \frac{-1 \pm \sqrt{1 + 2bVt}}{b}$ , except  $x > 0$ , and thus  $x = \frac{-1 + \sqrt{1 + 2bVt}}{b}$

(ii)  $Mf = \frac{d}{dt}(mv) = \frac{d}{dt}(M(1 + bx)v)$

So,  $ft + c' = (1 + bx)v$  and as  $= 0$ ,  $x = 0$ , and  $v = V$  we have  $c' = V$ .

Thus  $v = \frac{ft + V}{1 + bx}$  as required.

Separating variables and integrating

$\frac{1}{2}ft^2 + Vt + c'' = x + \frac{1}{2}bx^2$  and as  $x = 0$ , when  $t = 0$ ,  $c'' = 0$

So  $\frac{1}{2}bx^2 + x - \frac{1}{2}ft^2 - Vt = 0$ , and so  $x = \frac{-1 \pm \sqrt{1 + fbt^2 + 2bVt}}{b}$ , except  $x > 0$ , and thus  $x = \frac{-1 + \sqrt{1 + fbt^2 + 2bVt}}{b}$

If  $1 + fbt^2 + 2bVt$  is a perfect square, then  $x$  will be linear in  $t$  and  $\frac{dx}{dt}$  will be constant, i.e. if  $4b^2V^2 - 4fb = 0$ , that is  $bV^2 = f$

(in which case  $x = \frac{-1 + \sqrt{1 + b^2V^2t^2 + 2bVt}}{b} = \frac{-1 + (1 + bVt)}{b} = Vt$ , and  $v = V$  as expected.)

Otherwise,  $= \frac{ft + V}{1 + bx} = \frac{ft + V}{\sqrt{1 + fbt^2 + 2bVt}} = \frac{f + \frac{V}{t}}{\sqrt{fb + \frac{2bV}{t} + \frac{1}{t^2}}}$ , and as  $t \rightarrow \infty$ ,  $v \rightarrow \frac{f}{\sqrt{fb}} = \sqrt{\frac{f}{b}}$ ,

a constant, as required.



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