

STEP III, 2009 Q10 MS

10. Supposing that the particle P has mass m , the spring has natural length l , and modulus of elasticity λ , $mg = \frac{\lambda d}{l}$

If the speed of P when it hits the top of the spring is v , then $v = \sqrt{2gh}$

By Newton's second law, the second-order differential equation is thus

$$m\ddot{x} = mg - \frac{\lambda x}{l} = mg - \frac{mgx}{d} \text{ and so } \ddot{x} = g - \frac{gx}{d} \text{ with initial conditions that } x = 0,$$

$$\dot{x} = \sqrt{2gh}, \text{ when } t = 0.$$

$$\ddot{x} + \frac{gx}{d} = g \text{ has complementary function } x = B \cos \omega t + C \sin \omega t$$

where $\omega = \sqrt{\frac{g}{d}}$, and particular integral $x = A$, where $A = d$.

The initial conditions yield, $B = -d$ and $C = \sqrt{2dh}$

$$\text{So } x = d - d \cos \sqrt{\frac{g}{d}}t + \sqrt{2dh} \sin \sqrt{\frac{g}{d}}t.$$

$d \cos \sqrt{\frac{g}{d}}t - \sqrt{2dh} \sin \sqrt{\frac{g}{d}}t$ may be expressed in the form $R \cos \left(\sqrt{\frac{g}{d}}t + \alpha \right)$ where

$$R^2 = d^2 + 2dh, \text{ and } \tan \alpha = \frac{\sqrt{2dh}}{d} = \sqrt{\frac{2h}{d}}$$

$$\text{So } x = d - R \cos \left(\sqrt{\frac{g}{d}}t + \alpha \right)$$

$$x = 0 \text{ next when } t = T, \text{ that is when } 2\pi - \left(\sqrt{\frac{g}{d}}T + \alpha \right) = \alpha$$

$$\text{So } \sqrt{\frac{g}{d}}T = 2\pi - 2\alpha = 2\pi - 2 \tan^{-1} \sqrt{\frac{2h}{d}} \text{ and } T = \sqrt{\frac{d}{g}} \left(2\pi - 2 \tan^{-1} \sqrt{\frac{2h}{d}} \right).$$



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