

## STEP III, 2009, Q10

- 10 A light spring is fixed at its lower end and its axis is vertical. When a certain particle  $P$  rests on the top of the spring, the compression is  $d$ . When, instead,  $P$  is dropped onto the top of the spring from a height  $h$  above it, the compression at time  $t$  after  $P$  hits the top of the spring is  $x$ . Obtain a second-order differential equation relating  $x$  and  $t$  for  $0 \leq t \leq T$ , where  $T$  is the time at which  $P$  first loses contact with the spring.

Find the solution of this equation in the form

$$x = A + B \cos(\omega t) + C \sin(\omega t),$$

where the constants  $A$ ,  $B$ ,  $C$  and  $\omega$  are to be given in terms of  $d$ ,  $g$  and  $h$  as appropriate.

Show that

$$T = \sqrt{d/g} \left( 2\pi - 2 \arctan \sqrt{2h/d} \right).$$



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