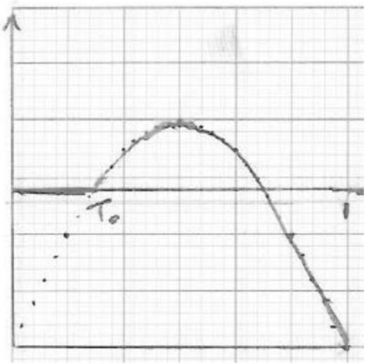
STEP III, 2008 Q9 MS

9. When the particle starts to move, friction is limiting and so
$mg\sin\pi T_0 - \mu mg = 0$
i.e. $\mu = \sin\pi T_0$

(i)

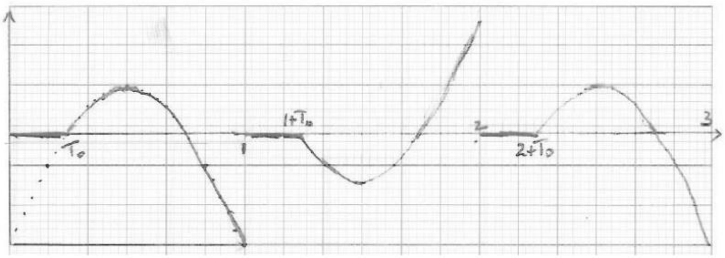


When the particle comes to rest, the area under the acceleration-time graph is zero

i.e. $\int_{T_0}^{1} g\sin\pi t - \mu_o g dt = 0$

Completing the manipulation and eliminating T_0 using the relation at the start of the question renders the required result.

(ii)



In the case $\mu = \mu_0$, the motion is periodic with period 2, the particle is stationary in intervals $(0, T_0), (1, 1+T_0), (2, 2+T_0)\ldots$, reversing its direction of motion after times 1, 2, 3, … , and returning to its starting point at time 2 (and 4,6….)

In the case $\mu = 0$, the motion is simple harmonic motion (period 2) superimposed on uniform motion , the particle instantaneously comes to rest at time 2, 4, …but otherwise always moves in the positive x direction.
($x = \frac{g}{\pi^2}(\pi t - \sin\pi t)$)