STEP III, 2008 Q9

9 A particle of mass m is initially at rest on a rough horizontal surface. The particle experiences a force $mg\sin\pi t$, where t is time, acting in a fixed horizontal direction. The coefficient of friction between the particle and the surface is μ. Given that the particle starts to move first at $t = T_0$, state the relation between T_0 and μ.

(i) For $\mu = \mu_0$, the particle comes to rest for the first time at $t = 1$. Sketch the acceleration-time graph for $0 \leqslant t \leqslant 1$. Show that

$$1 + \left(1 - \mu_0^2\right)^{\frac{1}{2}} - \mu_0\pi + \mu_0 \arcsin \mu_0 = 0\,.$$

(ii) For $\mu = \mu_0$ sketch the acceleration-time graph for $0 \leqslant t \leqslant 3$. Describe the motion of the particle in this case and in the case $\mu = 0$.

[**Note:** $\arcsin x$ is another notation for $\sin^{-1} x$.]