

STEP III, 2008 Q8 MS

8. (i) $p = -\frac{1}{2}$

$$(1 + px)S = \frac{1}{3}x \text{ with all other terms cancelling and so } S = \frac{1}{3}x / \left(1 - \frac{1}{2}x\right) = \frac{2x}{3(2-x)}$$

Using the sum of a GP

$$S_{n+1} = \frac{1}{3}x + \frac{1}{6}x^2 + \frac{1}{12}x^3 + \dots + \frac{1}{3 \times 2^n}x^{n+1} = \frac{\frac{1}{3}x \left(1 - \frac{x^{n+1}}{2^{n+1}}\right)}{\left(1 - \frac{x}{2}\right)}$$

Alternatively $S_{n+1} = S - (a_{n+2}x^{n+2} + \dots) = S - \frac{1}{2^{n+1}}x^{n+1}S$

$$= \left(1 - \frac{x^{n+1}}{2^{n+1}}\right) \frac{2x}{3(2-x)}$$

(ii) Using similar working to part (i)

$$18 + 8p + 2q = 0$$

$$37 + 18p + 8q = 0$$

$$\text{so } p = -\frac{5}{2}, q = 1$$

$$\text{and so } (1 + px + qx^2)T = 2 + 3x$$

$$\text{giving } T = (2 + 3x) / \left(1 - \frac{5}{2}x + x^2\right) = \frac{4 + 6x}{2 - 5x + 2x^2} = \frac{4 + 6x}{(2-x)(1-2x)}$$

$$\text{By partial fractions } T = \frac{14}{3}(1-2x)^{-1} - \frac{8}{3}\left(1 - \frac{x}{2}\right)^{-1}$$

$$\text{and so } T_{n+1} = \frac{14}{3}(1 + 2x + (2x)^2 + \dots + (2x)^n) - \frac{8}{3}\left(1 + \frac{x}{2} + \left(\frac{x}{2}\right)^2 + \dots + \left(\frac{x}{2}\right)^n\right)$$

$$= \frac{14}{3} \frac{(1 - (2x)^{n+1})}{1 - 2x} - \frac{8}{3} \frac{\left(1 - \left(\frac{x}{2}\right)^{n+1}\right)}{1 - \frac{x}{2}}$$



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