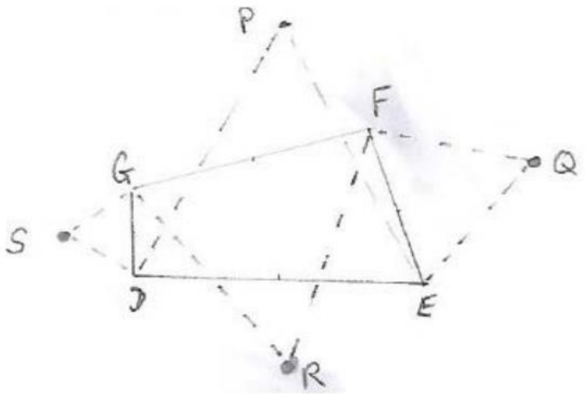
STEP III, 2008 Q7 MS

7. The starting point $c-a=\frac{1}{2}\left(1+i\sqrt{3}\right)(b-a)$ leads to the given result.

Interchanging a and b gives $2c=(a+b)+i\sqrt{3}(a-b)$ if A, B, C are described clockwise.

(i) The clue to this is the phrase "can be chosen" and a sketch demonstrates that a pair of the equilateral triangles need to be clockwise, and the other pair anti-clockwise



Applying the results in the stem of the question to this configuration,

$$2p=(d+e)+i\sqrt{3}(e-d)$$
$$2q=(e+f)+i\sqrt{3}(e-f)$$
$$2r=(f+g)+i\sqrt{3}(g-f)$$
$$2s=(g+d)+i\sqrt{3}(g-d)$$

and so $2PS=(g-e)+i\sqrt{3}(g-e)=-2RQ$, PSQR is a parallelogram.
(The pairs could have been chosen with opposite parity leading to very similar working.)

(ii) Supposing LMN is clockwise, U is the centroid of equilateral triangle LMH, V of MNJ, and W of NLK, then
$3u=l+m+h$ where $2h=(l+m)+i\sqrt{3}(m-l)$ with similar results for v and w.

Both $6w$, and $3\left[(u+v)+i\sqrt{3}(u-v)\right]$ can be shown to equal $3(n+l)+i\sqrt{3}(l-n)$ and so UVW is a clockwise equilateral triangle.