

STEP III, 2008 Q7

- 7 The points A , B and C in the Argand diagram are the vertices of an equilateral triangle described anticlockwise. Show that the complex numbers a , b and c representing A , B and C satisfy

$$2c = (a + b) + i\sqrt{3}(b - a).$$

Find a similar relation in the case that A , B and C are the vertices of an equilateral triangle described clockwise.

- (i) The quadrilateral $DEFG$ lies in the Argand diagram. Show that points P , Q , R and S can be chosen so that PDE , QEF , RFG and SGD are equilateral triangles and $PQRS$ is a parallelogram.
- (ii) The triangle LMN lies in the Argand diagram. Show that the centroids U , V and W of the equilateral triangles drawn externally on the sides of LMN are the vertices of an equilateral triangle.

[**Note:** The *centroid* of a triangle with vertices represented by the complex numbers x , y and z is the point represented by $\frac{1}{3}(x + y + z)$.]



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