STEP III, 2008 Q7

7 The points A, B and C in the Argand diagram are the vertices of an equilateral triangle described anticlockwise. Show that the complex numbers a, b and c representing A, B and C satisfy

$$2c = (a+b) + i\sqrt{3}(b-a).$$

Find a similar relation in the case that A, B and C are the vertices of an equilateral triangle described clockwise.

(i) The quadrilateral $DEFG$ lies in the Argand diagram. Show that points P, Q, R and S can be chosen so that PDE, QEF, RFG and SGD are equilateral triangles and $PQRS$ is a parallelogram.

(ii) The triangle LMN lies in the Argand diagram. Show that the centroids U, V and W of the equilateral triangles drawn externally on the sides of LMN are the vertices of an equilateral triangle.

[**Note:** The *centroid* of a triangle with vertices represented by the complex numbers x, y and z is the point represented by $\frac{1}{3}(x+y+z)$.]