

## STEP III, 2008 Q6 MS

6. (i) Differentiating  $y = p^2 + 2xp$  with respect to  $x$  gives  

$$p = 2p \frac{dp}{dx} + 2x \frac{dp}{dx} + 2p$$
 which can be rearranged suitably.  
 The differential equation  $\frac{dx}{dp} + \frac{2}{p}x = -2$  has an integrating factor  $p^2$   
 and integrating will give the required general solution.  
 Substituting  $x = 2, p = -3$ , leads to  $A = 0$ , i.e.  $p = -\frac{3}{2}x$  which can be  
 substituted in the original equation and so  $y = -\frac{3}{4}x^2$ .
- (ii) The same approach as in part (i) generates  $\frac{dx}{dp} + \frac{2}{p}x = -\frac{(\ln p + 1)}{p}$ ,  
 which with the same integrating factor has general solution  

$$x = -\frac{1}{4} - \frac{1}{2} \ln p + Bp^{-2}$$
 and particular solution  

$$x = -\frac{1}{2} \ln p - \frac{1}{4}$$

Again, substitution of  $\ln p$  (and  $p$ ) in the original equation leads to the solution  
 which is  $y = -\frac{1}{2}e^{-2x-\frac{1}{2}}$



# NextStepMaths.com

To view mark schemes, fully worked solutions and  
 examiner's comments, and for more details about  
 tutoring and other services offered, go to  
[NextStepMaths.com](http://NextStepMaths.com)