STEP III, 2008 Q6 MS

6. (i) Differentiating $y = p^2 + 2xp$ with respect to x gives

$p = 2p\frac{dp}{dx} + 2x\frac{dp}{dx} + 2p$ which can be rearranged suitably.

The differential equation $\frac{dx}{dp} + \frac{2}{p}x = -2$ has an integrating factor p^2 and integrating will give the required general solution.

Substituting $x = 2, p = -3$, leads to $A = 0$, i.e. $p = -\frac{3}{2}x$ which can be substituted in the original equation and so $y = -\frac{3}{4}x^2$.

(ii) The same approach as in part (i) generates $\frac{dx}{dp} + \frac{2}{p}x = -\frac{(\ln p + 1)}{p}$,

which with the same integrating factor has general solution

$x = -\frac{1}{4} - \frac{1}{2}\ln p + Bp^{-2}$

and particular solution

$x = -\frac{1}{2}\ln p - \frac{1}{4}$

Again, substitution of ln p (and p) in the original equation leads to the solution which is $y = -\frac{1}{2}e^{-2x-\frac{1}{2}}$