

## STEP III, 2008 Q5

- 5 The functions  $T_n(x)$ , for  $n = 0, 1, 2, \dots$ , satisfy the recurrence relation

$$T_{n+1}(x) - 2xT_n(x) + T_{n-1}(x) = 0 \quad (n \geq 1). \quad (*)$$

Show by induction that

$$(T_n(x))^2 - T_{n-1}(x)T_{n+1}(x) = f(x),$$

where  $f(x) = (T_1(x))^2 - T_0(x)T_2(x)$ .

In the case  $f(x) \equiv 0$ , determine (with proof) an expression for  $T_n(x)$  in terms of  $T_0(x)$  (assumed to be non-zero) and  $r(x)$ , where  $r(x) = T_1(x)/T_0(x)$ . Find the two possible expressions for  $r(x)$  in terms of  $x$ .



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