

STEP III, 2008 Q3 MS

$$3. \quad \frac{dy}{dx} = \frac{b \cos \theta}{-a \sin \theta}$$

So the line ON is $y = \frac{a \sin \theta}{b \cos \theta} x$

SP is $y = \frac{b \sin \theta}{a(\cos \theta + e)}(x + ae)$

Solving simultaneously by substituting for x to find the y coordinate of T ,

$$y = \frac{b \sin \theta}{a(\cos \theta + e)} \left(\frac{b \cos \theta}{a \sin \theta} y + ae \right)$$

and using $b^2 = a^2(1 - e^2)$ to eliminate a^2 gives the required result.

Then the x coordinate of T is $\frac{b^2 \cos \theta}{a(1 + e \cos \theta)}$.

Eliminating θ using $\sec \theta + e = \frac{b^2}{ax}$ and $\tan \theta = \frac{by}{ax}$,

$$(x, y) \text{ satisfies } \left(\frac{b^2}{ax} - e \right)^2 = 1 + \left(\frac{by}{ax} \right)^2$$

and again using $b^2 = a^2(1 - e^2)$, this time to eliminate b^2 , gives, following simplifying algebra

$$(x + ae)^2 + y^2 = a^2, \text{ as required.}$$



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