

## STEP III, 2008 Q2

- 2 Let  $S_k(n) \equiv \sum_{r=0}^n r^k$ , where  $k$  is a positive integer, so that

$$S_1(n) \equiv \frac{1}{2}n(n+1) \quad \text{and} \quad S_2(n) \equiv \frac{1}{6}n(n+1)(2n+1).$$

- (i) By considering  $\sum_{r=0}^n [(r+1)^k - r^k]$ , show that

$$kS_{k-1}(n) = (n+1)^k - (n+1) - \binom{k}{2}S_{k-2}(n) - \binom{k}{3}S_{k-3}(n) - \cdots - \binom{k}{k-1}S_1(n). \quad (*)$$

Obtain simplified expressions for  $S_3(n)$  and  $S_4(n)$ .

- (ii) Explain, using (\*), why  $S_k(n)$  is a polynomial of degree  $k+1$  in  $n$ . Show that in this polynomial the constant term is zero and the sum of the coefficients is 1.



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