

## STEP III, 2008 Q1 MS

1. Following the hint yields

$$ax^2 + by^2 + (a+b)xy = \frac{1}{3}(x+y)$$

which is  $\frac{1}{5} + xy = \frac{1}{3}(x+y)$

The same trick applied to the third equation gives  $\frac{1}{7} + \frac{1}{3}xy = \frac{1}{5}(x+y)$ .

The two equations can be solved simultaneously for  $xy$  and  $(x+y)$ , giving

$$xy = \frac{3}{35} \text{ and } (x+y) = \frac{6}{7}$$

Thus  $x$  and  $y$  are the roots of the quadratic equation  $35z^2 - 30z + 3 = 0$   
( $x$  and  $y$  are interchangeable).

$a$  and  $b$  are then found by substituting back into two of the original equations and the full solution is

$$x = \frac{3}{7} \pm \frac{2}{35}\sqrt{30} = \frac{3}{7} \pm \frac{2}{7}\sqrt{\frac{6}{5}}$$

$$y = \frac{3}{7} \mp \frac{2}{35}\sqrt{30} = \frac{3}{7} \mp \frac{2}{7}\sqrt{\frac{6}{5}}$$

$$a = \frac{1}{2} \mp \frac{\sqrt{30}}{36} = \frac{1}{2} \mp \frac{1}{6}\sqrt{\frac{5}{6}}$$

$$b = \frac{1}{2} \pm \frac{\sqrt{30}}{36} = \frac{1}{2} \pm \frac{1}{6}\sqrt{\frac{5}{6}}$$



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