

STEP III, 2008 Q13 MS

$$13. \quad P(1 \text{ ring created at first step}) = \frac{1}{2n-1},$$

$$P(0 \text{ rings created at first step}) = \frac{2n-2}{2n-1}$$

$$E(\text{number of rings created at first step}) = \frac{1}{2n-1} \times 1 + \frac{2n-2}{2n-1} \times 0 = \frac{1}{2n-1}$$

Regardless of what happens at first step, after the first step there $2n-2$ free ends. Similarly after second step $2n-4$ free ends regardless, etc.

$$E(\text{number of rings at end of process}) = \frac{1}{2n-1} + \frac{1}{2n-3} + \frac{1}{2n-5} + \frac{1}{2n-7} + \dots + \frac{1}{1}$$

$Var(\text{number of rings at end of process}) =$

$$\frac{1}{2n-1} - \left(\frac{1}{2n-1}\right)^2 + \frac{1}{2n-3} - \left(\frac{1}{2n-3}\right)^2 + \frac{1}{2n-5} - \left(\frac{1}{2n-5}\right)^2 + \frac{1}{2n-7} - \left(\frac{1}{2n-7}\right)^2 + \dots + \frac{1}{1} - \left(\frac{1}{1}\right)^2$$

(as numbers of rings created at each step are independent)

$$= \frac{2(n-1)}{(2n-1)^2} + \frac{2(n-2)}{(2n-3)^2} + \frac{2(n-3)}{(2n-5)^2} + \dots + \frac{2}{3^2}$$

$$\text{For } n = 40000, E(\text{number of rings created}) = 1 + \frac{1}{3} + \frac{1}{5} + \dots + \frac{1}{79999}$$

$$= 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \dots + \frac{1}{80000} - \left(\frac{1}{2} + \frac{1}{4} + \dots + \frac{1}{80000}\right)$$

$$\approx \ln 80000 - \frac{1}{2} \ln 40000$$

$$= 2 \ln 20$$

$$\approx 6$$



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