

STEP III, 2008 Q13

- 13 A box contains n pieces of string, each of which has two ends. I select two string ends at random and tie them together. This creates either a ring (if the two ends are from the same string) or a longer piece of string. I repeat the process of tying together string ends chosen at random until there are none left.

Find the expected number of rings created at the first step and hence obtain an expression for the expected number of rings created by the end of the process. Find also an expression for the variance of the number of rings created.

Given that $\ln 20 \approx 3$ and that $1 + \frac{1}{2} + \dots + \frac{1}{n} \approx \ln n$ for large n , determine approximately the expected number of rings created in the case $n = 40\,000$.



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