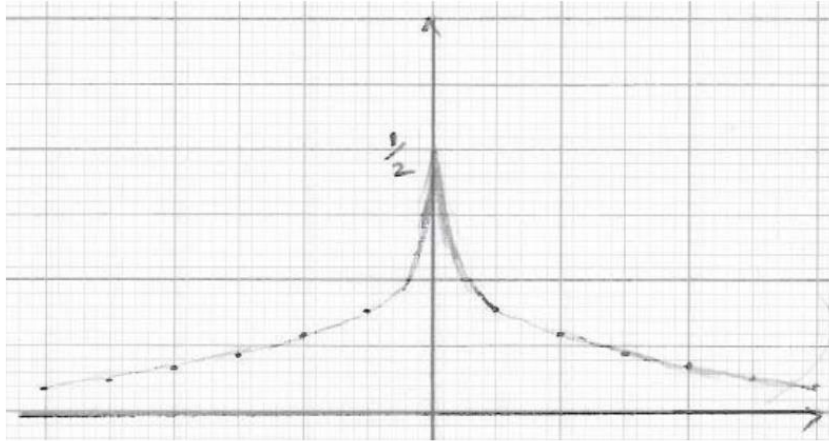


## STEP III, 2008 Q12 MS

12.



$$\begin{aligned}
 M_x(\theta) &= \int_{-\infty}^{\infty} e^{\theta x} f(x) dx = \int_{-\infty}^0 \frac{1}{2} e^{x(1+\theta)} dx + \int_0^{\infty} \frac{1}{2} e^{-x(1-\theta)} dx \\
 &= \frac{1}{2(1+\theta)} \left[ e^{x(1+\theta)} \right]_{-\infty}^0 - \frac{1}{2(1-\theta)} \left[ e^{-x(1-\theta)} \right]_0^{\infty} = \frac{1}{2(1+\theta)} + \frac{1}{2(1-\theta)} \quad (\text{requiring } |\theta| < 1) \\
 &= (1-\theta^2)^{-1}
 \end{aligned}$$

$$\begin{aligned}
 \text{Var}(X) &= M_x''(0) - (M_x'(0))^2 \\
 M_x'(\theta) &= 2\theta(1-\theta^2)^{-2}, \quad M_x''(\theta) = 2(1-\theta^2)^{-2} + 8\theta(1-\theta^2)^{-3} \\
 \text{and so } M_x'(0) &= 0, \quad M_x''(0) = 2, \quad \text{Var}(X) = 2
 \end{aligned}$$

$$\begin{aligned}
 \text{Or alternatively, } M_x(\theta) &= E(e^{\theta X}) = E\left(1 + \theta X + \frac{1}{2}\theta^2 X^2 + \dots\right) \\
 &= (1-\theta^2)^{-1} = 1 + \theta^2 + \theta^4 + \dots
 \end{aligned}$$

$$\text{and so } E(X) = 0, \quad E\left(\frac{1}{2}X^2\right) = 1, \quad \text{Var}(X) = E(X^2) - (E(X))^2 = 2$$

$$\text{If } T = Y/\sqrt{2n}, \text{ then } M_T(\theta) = E(e^{\theta T}) = E\left(e^{\theta \sum (x_i/\sqrt{2n})}\right) = \prod_{i=1}^n E\left(e^{\frac{\theta}{\sqrt{2n}} X_i}\right) = \left(1 - \frac{\theta^2}{2n}\right)^{-n}$$



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$$\log(M_T(\theta)) = -n \log\left(1 - \frac{\theta^2}{2n}\right) = -n \left[ -\frac{\theta^2}{2n} - \frac{\theta^4}{8n^2} - \frac{\theta^6}{24n^3} - \dots \right] = \frac{\theta^2}{2} + \frac{\theta^4}{8n} + \frac{\theta^6}{24n^2} + \dots$$

Thus as  $n \rightarrow \infty$ ,  $\log(M_T(\theta)) \rightarrow \frac{\theta^2}{2}$ , and so  $M_T(\theta) \rightarrow \exp\left(\frac{\theta^2}{2}\right)$

$$P(|Y| \geq 25) = 0.05 \quad \text{and} \quad P\left(|Y/\sqrt{2n}| \geq 1.96\right) = 0.05 \quad \text{and so}$$

$$25 = 1.96\sqrt{2n}$$

$$2n = \frac{25^2}{1.96^2} \approx \frac{625}{4}$$

$$n \approx \frac{625}{8} \approx 78$$



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