

STEP III, 2008 Q12

- 12 Let X be a random variable with a Laplace distribution, so that its probability density function is given by

$$f(x) = \frac{1}{2}e^{-|x|}, \quad -\infty < x < \infty. \quad (*)$$

Sketch $f(x)$. Show that its moment generating function $M_X(\theta)$ is given by $M_X(\theta) = (1 - \theta^2)^{-1}$ and hence find the variance of X .

A frog is jumping up and down, attempting to land on the same spot each time. In fact, in each of n successive jumps he always lands on a fixed straight line but when he lands from the i th jump ($i = 1, 2, \dots, n$) his displacement from the point from which he jumped is X_i cm, where X_i has the distribution $(*)$. His displacement from his starting point after n jumps is Y cm (so that $Y = \sum_{i=1}^n X_i$). Each jump is independent of the others.

Obtain the moment generating function for $Y/\sqrt{2n}$ and, by considering its logarithm, show that this moment generating function tends to $\exp(\frac{1}{2}\theta^2)$ as $n \rightarrow \infty$.

Given that $\exp(\frac{1}{2}\theta^2)$ is the moment generating function of the standard Normal random variable, estimate the least number of jumps such that there is a 5% chance that the frog lands 25 cm or more from his starting point.



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