STEP III, 2008 Q11 MS

11. If the resistance couple (constant) is L, then using $L = I\alpha$ for the second phase of the motion, $L = \frac{I\omega_0}{T}$ and rotational kinetic energy used up doing work against the couple in the second phase gives

$$\frac{1}{2} I\omega_0^{\,2} = L \times n_2 \times 2\pi$$

Hence, eliminating L and simplifying gives the first result.

If the particle descends a distance h in the first phase of motion, then $h = 2\pi r n_1$. If the particle has speed v at the end of the first phase, then $v = r\omega_0$
and using the work-energy principle,

$$mgh - L \times n_1 \times 2\pi = \frac{1}{2} I\omega_0^{\,2} + \frac{1}{2} mv^2$$

Hence, eliminating h, v and $\omega_0^{\,2}$ obtains the second result.