

## STEP III, 2008 Q10 MS

10. Considering the  $r$ th short string  $T_r = mg + T_{r-1}$

Also we have  $T_r = \frac{\lambda x_r}{l}$ , and  $T_1 = mg$

Thus  $T_r = rmg$  and so the total length is given by  $\sum_1^n (l + x_r) = \sum_1^n \left( l + \frac{rmgl}{\lambda} \right)$   
 $= nl + \frac{mgl}{\lambda} \frac{n(n+1)}{2}$

The elastic energy stored is  $\sum_1^n \frac{\lambda x_r^2}{2l} = \sum_1^n \lambda \left( \frac{lmg}{\lambda} \right)^2 \frac{r^2}{2l} = \frac{m^2 g^2 l}{12\lambda} n(n+1)(2n+1)$

For the uniform heavy rope, we let  $M = nm$ ,  $L_0 = nl$ , and consider the limit as  $n \rightarrow \infty$

$L = \lim \left( L_0 + \frac{M}{n} \frac{g}{\lambda} \frac{L_0}{n} \frac{n(n+1)}{2} \right) = L_0 \left( 1 + \frac{Mg}{2\lambda} \right)$

and the elastic energy stored is

$\lim \left( \frac{m^2 g^2 l}{12\lambda} n(n+1)(2n+1) \right) = \lim \left( \frac{M^2 g^2 L_0}{12\lambda} \frac{n(n+1)(2n+1)}{n^3} \right) = \frac{M^2 g^2 L_0}{6\lambda}$

and eliminating  $M$  using the result just found for  $L$  we obtain  $\frac{2\lambda(L - L_0)^2}{3L_0}$



# NextStepMaths.com

To view mark schemes, fully worked solutions and examiner's comments, and for more details about tutoring and other services offered, go to [NextStepMaths.com](http://NextStepMaths.com)