*Attempt this question independently, showing full and clear working.*

**STEP III 2008 Question 10**

A long string consists of $n$ short light strings joined together, each of natural length $\ell$ and modulus of elasticity $\lambda$. It hangs vertically at rest, suspended from one end. Each of the short strings has a particle of mass $m$ attached to its lower end. The short strings are numbered 1 to $n$, the $n$th short string being at the top. By considering the tension in the $r$th short string, determine the length of the long string. Find also the elastic energy stored in the long string.

A uniform heavy rope of mass $M$ and natural length $L_0$ has modulus of elasticity $\lambda$. The rope hangs vertically at rest, suspended from one end. Show that the length, $L$, of the rope is given by

$$L = L_0\left(1 + \frac{Mg}{2\lambda}\right),$$

and find an expression in terms of $L$, $L_0$ and $\lambda$ for the elastic energy stored in the rope.