

STEP III, 2007 Q8 MS

8. (i) Substituting each u into the differential equation yields simultaneous equations $a(x) + xb(x) = 0$ and $e^{-x}(1 - a(x) + b(x)) = 0$ which solve to give

$$a(x) = \frac{x}{1+x} \text{ and } b(x) = \frac{-1}{1+x}$$

The general solution is $u = Ax + Be^{-x}$.

$y = \frac{1}{3u} \frac{du}{dx} \Rightarrow \frac{dy}{dx} = \frac{-1}{3u^2} \left(\frac{du}{dx} \right)^2 + \frac{1}{3u} \frac{d^2u}{dx^2}$ which when substituted into equation (*), multiplied by $3u$, and collected on one side gives the required result.

$$u = Ax + Be^{-x} \Rightarrow \frac{du}{dx} = A - Be^{-x} \Rightarrow y = \frac{A - Be^{-x}}{3(Ax + Be^{-x})},$$

and substitution of $x = 0, y = 0$ gives $A = B$ and hence $y = \frac{1 - e^{-x}}{3(x + e^{-x})}$.

(ii) Substituting $y = \frac{1}{u} \frac{du}{dx}$ into the given equation yields

$$\frac{d^2u}{dx^2} + \frac{x}{1-x} \frac{du}{dx} - \frac{1}{1-x} u = 0 \text{ which is the equation in the first part with } x \text{ replaced by } -x$$

So the general solution is $u = Cx + De^x$

Substitution of $x = 0, y = 2$ again gives $A = B$, and hence $y = \frac{1 + e^x}{x + e^x}$



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