

STEP III, 2007, Q8

- 8 (i) Find functions $a(x)$ and $b(x)$ such that $u = x$ and $u = e^{-x}$ both satisfy the equation

$$\frac{d^2u}{dx^2} + a(x)\frac{du}{dx} + b(x)u = 0.$$

For these functions $a(x)$ and $b(x)$, write down the general solution of the equation.

Show that the substitution $y = \frac{1}{3u} \frac{du}{dx}$ transforms the equation

$$\frac{dy}{dx} + 3y^2 + \frac{x}{1+x}y = \frac{1}{3(1+x)} \quad (*)$$

into

$$\frac{d^2u}{dx^2} + \frac{x}{1+x} \frac{du}{dx} - \frac{1}{1+x}u = 0$$

and hence show that the solution of equation (*) that satisfies $y = 0$ at $x = 0$ is given

$$\text{by } y = \frac{1 - e^{-x}}{3(x + e^{-x})}.$$

- (ii) Find the solution of the equation

$$\frac{dy}{dx} + y^2 + \frac{x}{1-x}y = \frac{1}{1-x}$$

that satisfies $y = 2$ at $x = 0$.



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