

STEP III, 2007 Q6 MS

6. $pp^* = qq^* = a^2$

and so $a^2(p - q) = qq^*p - pp^*q = -pq(p^* - q^*)$ and hence the required result.

If PQ and RS are perpendicular then $p - q = ki(r - s)$ for some real k , and thus

$$p^* - q^* = -ki(r^* - s^*), \text{ and so } pq = -a^2 \frac{p - q}{p^* - q^*} = a^2 \frac{r - s}{r^* - s^*} = -rs$$

For $n = 3$, $B_1B_2 \perp A_1A_2$ etc. $\Rightarrow a_1a_2 + b_1b_2 = 0$ etc.

$$\text{Thus } b_1^2 = \frac{b_1b_2 \times b_1b_3}{b_2b_3} = \frac{-a_1a_2 \times -a_1a_3}{-a_2a_3} = -a_1^2 \text{ and so } b_1 = \pm ia_1$$

i.e. two choices of B_1 .

For $n = 4$, $B_1B_2 \perp A_1A_2$ etc. $\Rightarrow a_1a_2 + b_1b_2 = 0$ etc. but this only yields 3 independent equations as e.g. $a_3a_4 + b_3b_4 = 0$ can be obtained from the other three equations by

$$a_3a_4 = \frac{a_2a_3 \times a_4a_1}{a_2a_1} \text{ etc. Hence there are arbitrarily many possible choices for } B_1.$$

For $n > 4$, the corresponding results are as for $n = 3$ or $n = 4$ depending on whether n is odd or even.



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