

## STEP III, 2007, Q5

- 5 Let  $y = \ln(x^2 - 1)$ , where  $x > 1$ , and let  $r$  and  $\theta$  be functions of  $x$  determined by  $r = \sqrt{x^2 - 1}$  and  $\coth \theta = x$ . Show that

$$\frac{dy}{dx} = \frac{2 \cosh \theta}{r} \quad \text{and} \quad \frac{d^2y}{dx^2} = -\frac{2 \cosh 2\theta}{r^2},$$

and find an expression in terms of  $r$  and  $\theta$  for  $\frac{d^3y}{dx^3}$ .

Find, with proof, a similar formula for  $\frac{d^ny}{dx^n}$  in terms of  $r$  and  $\theta$ .



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