

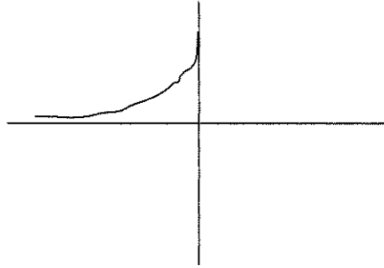
STEP III, 2007 Q4 MS

4.

$$y = a \sin t \Rightarrow \dot{y} = a \cos t$$

$$x = a \left(\cos t + \ln \tan \frac{t}{2} \right) \Rightarrow \dot{x} = a \left(-\sin t + \frac{\frac{1}{2} \sec^2 \frac{t}{2}}{\tan \frac{t}{2}} \right) = a(-\sin t + \operatorname{cosec} t) = a \cos t \cot t$$

giving $\frac{dy}{dx} = \tan t$.



(y intercept a, y axis tangential to curve, x axis asymptote)

Tangent is $y - a \sin t = \tan t \left(x - a \left(\cos t + \ln \tan \frac{t}{2} \right) \right)$ giving Q as $\left(a \ln \tan \frac{t}{2}, 0 \right)$ and

$$\text{thus } PQ = \sqrt{(a \cos t)^2 + (a \sin t)^2} = a$$

$$\dot{y} = a \cos t \Rightarrow \ddot{y} = -a \sin t$$

$$\dot{x} = a(-\sin t + \operatorname{cosec} t) \Rightarrow \ddot{x} = a(-\cos t - \operatorname{cosec} t \cot t)$$

$$\dot{x}^2 + \dot{y}^2 = (a \cos t \cot t)^2 + (a \cos t)^2 = a^2 \cot^2 t$$

$$\begin{aligned} x \ddot{y} - \dot{y} \ddot{x} &= a \cos t \cot t \times -a \sin t - a \cos t \times a(-\cos t - \operatorname{cosec} t \cot t) \\ &= a^2(-\cos^2 t + \cos^2 t + \cot^2 t) = a^2 \cot^2 t \end{aligned}$$

giving $\rho = a \cot t$.

From the results for $\frac{dy}{dx}$ and ρ , C is

$$\left(a \left(\cos t + \ln \tan \frac{t}{2} \right) - \rho \sin t, a \sin t + \rho \cos t \right) = \left(a \ln \tan \frac{t}{2}, a \operatorname{cosec} t \right)$$

Which has the same x coordinate as Q.



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