

STEP III, 2007, Q4

- 4 A curve is given parametrically by

$$\begin{aligned}x &= a\left(\cos t + \ln \tan \frac{1}{2}t\right), \\y &= a \sin t,\end{aligned}$$

where $0 < t < \frac{1}{2}\pi$ and a is a positive constant. Show that $\frac{dy}{dx} = \tan t$ and sketch the curve.

Let P be the point with parameter t and let Q be the point where the tangent to the curve at P meets the x -axis. Show that $PQ = a$.

The radius of curvature, ρ , at P is defined by

$$\rho = \frac{(\dot{x}^2 + \dot{y}^2)^{\frac{3}{2}}}{|\dot{x}\ddot{y} - \dot{y}\ddot{x}|},$$

where the dots denote differentiation with respect to t . Show that $\rho = a \cot t$.

The point C lies on the normal to the curve at P , a distance ρ from P and above the curve. Show that CQ is parallel to the y -axis.



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