

STEP III, 2007, Q3

- 3 A sequence of numbers, F_1, F_2, \dots , is defined by $F_1 = 1, F_2 = 1$, and

$$F_n = F_{n-1} + F_{n-2} \quad \text{for } n \geq 3.$$

- (i) Write down the values of F_3, F_4, \dots, F_8 .
- (ii) Prove that $F_{2k+3}F_{2k+1} - F_{2k+2}^2 = -F_{2k+2}F_{2k} + F_{2k+1}^2$.
- (iii) Prove by induction or otherwise that $F_{2n+1}F_{2n-1} - F_{2n}^2 = 1$ and deduce that $F_{2n}^2 + 1$ is divisible by F_{2n+1} .
- (iv) Prove that $F_{2n-1}^2 + 1$ is divisible by F_{2n+1} .



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