

## STEP III, 2007 Q2 MS

2. (i)

$$1.3.5.7\dots(2n-1) = \frac{1.2.3.4\dots 2n}{2.4.6.8\dots 2n} = \frac{(2n)!}{2.1.2.2.2.3.2.4\dots 2.n} = \frac{(2n)!}{2^n \cdot 1.2.3.4\dots n} = \frac{(2n)!}{2^n n!}$$

Using the binomial theorem, which is valid given the condition  $|x| < \frac{1}{4}$ ,

$$\begin{aligned} (1-4x)^{-\frac{1}{2}} &= 1 + \frac{-1}{2}(-4x) + \frac{\frac{-1-3}{2}(-4x)^2}{2!} + \dots \\ &= 1 + 1.(2x) + \frac{1.3}{2!}(2x)^2 + \dots + \frac{1.3.5.7\dots(2n-1)}{n!}(2x)^n + \dots \end{aligned}$$

So the first result of the question yields  $(1-4x)^{-\frac{1}{2}} = 1 + \sum_{n=1}^{\infty} \frac{(2n)!}{2^n n!} \frac{(2x)^n}{n!}$  leading to the required expression.

(ii) Differentiating  $(1-4x)^{-\frac{1}{2}} = 1 + \sum_{n=1}^{\infty} \frac{(2n)!x^n}{(n!)^2}$  with respect to  $x$ , and multiplying the result by  $x$  gives  $\frac{2x}{(1-4x)^{\frac{3}{2}}} = \sum_{n=1}^{\infty} \frac{(2n)!x^n}{n!(n-1)!}$  and substituting  $x = \frac{6}{25} < \frac{1}{4}$ , gives the desired result.

(iii) Integrating  $(1-4x)^{-\frac{1}{2}} = 1 + \sum_{n=1}^{\infty} \frac{(2n)!x^n}{(n!)^2}$  with respect to  $x$ , gives  $\frac{-1}{2}(1-4x)^{\frac{1}{2}} = x + \sum_{n=1}^{\infty} \frac{(2n)!x^{n+1}}{(n+1)!n!} + c$ , and substituting  $x = 0 < \frac{1}{4}$ , gives  $c = \frac{-1}{2}$ .

Now substituting  $x = \frac{2}{9} = \frac{2}{3^2} < \frac{1}{4}$  and simplifying, gives the desired result.



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