

STEP III, 2007 Q12 MS

12.

$$E(N) = \sum_{i=1}^{2n-1} \frac{1}{2n-1} i = \frac{1}{2n-1} \frac{(2n-1)2n}{2} = n$$

$$E(N^2) = \sum_{i=1}^{2n-1} \frac{1}{2n-1} i^2 = \frac{1}{2n-1} \frac{(2n-1)2n(4n-1)}{6} = \frac{n(4n-1)}{3}$$

$$E(Y) = E\left(\sum_{i=1}^N X_i\right) = \frac{1}{2n-1} E(X_1) + \frac{1}{2n-1} E(X_1 + X_2) + \dots = \frac{1}{2n-1} (\mu + 2\mu + 3\mu + \dots + (2n-1)\mu)$$

$$= \frac{1}{2n-1} \frac{\mu(2n-1)2n}{2} = n\mu$$

$$E(YN) = \frac{1}{2n-1} \times 1 \times \mu + \frac{1}{2n-1} \times 2 \times 2\mu + \dots + \frac{1}{2n-1} \times (2n-1) \times (2n-1)\mu = \frac{n(4n-1)}{3} \mu$$

and so $Cov(Y, N) = \frac{n(4n-1)}{3} \mu - n^2 \mu = \frac{1}{3} n(n-1) \mu$

$$E(X_i^2) = Var(X_i) + (E(X_i))^2 = \sigma^2 + \mu^2$$

Also $(X_1 + X_2 + \dots + X_r)^2 = \sum_{i=1}^r X_i^2 + 2 \sum_{i \neq j} X_i X_j$, and so

$$E\left((X_1 + X_2 + \dots + X_r)^2\right) = r(\sigma^2 + \mu^2) + 2 \frac{r(r-1)}{2} \mu^2$$

Thus

$$E(Y^2) = \frac{1}{2n-1} \sum_{r=1}^{2n-1} \left(r(\sigma^2 + \mu^2) + 2 \frac{r(r-1)}{2} \mu^2 \right) = n(\sigma^2 + \mu^2) + \frac{n(4n-1)}{3} \mu^2 - n\mu^2 = n\sigma^2 + \frac{n(4n-1)}{3} \mu^2$$

and so $Var(Y) = n\sigma^2 + \frac{n(4n-1)}{3} \mu^2 - n^2 \mu^2 = n\sigma^2 + \frac{n(n-1)}{3} \mu^2$



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