

## STEP III, 2007, Q12

- 12 I choose a number from the integers  $1, 2, \dots, (2n - 1)$  and the outcome is the random variable  $N$ . Calculate  $E(N)$  and  $E(N^2)$ .

I then repeat a certain experiment  $N$  times, the outcome of the  $i$ th experiment being the random variable  $X_i$  ( $1 \leq i \leq N$ ). For each  $i$ , the random variable  $X_i$  has mean  $\mu$  and variance  $\sigma^2$ , and  $X_i$  is independent of  $X_j$  for  $i \neq j$  and also independent of  $N$ . The random variable  $Y$  is defined by  $Y = \sum_{i=1}^N X_i$ . Show that  $E(Y) = n\mu$  and that  $\text{Cov}(Y, N) = \frac{1}{3}n(n-1)\mu$ . Find  $\text{Var}(Y)$  in terms of  $n, \sigma^2$  and  $\mu$ .



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