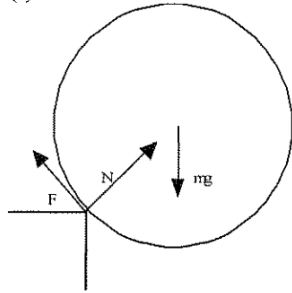


STEP III, 2007 Q11 MS

11. (i)



If the angle between mg and N is θ , then conserving energy and either differentiating the energy equation or taking moments about the point of contact yields

$$\frac{1}{2}mu^2 + mga = \frac{1}{2}ma^2\dot{\theta}^2 + mga\cos\theta \quad \text{and} \quad 0 = a\ddot{\theta} - g\sin\theta$$

Resolving in the opposite direction to F , $mg\sin\theta - F = ma\ddot{\theta}$ and so, from the second equation above, $F = 0$.

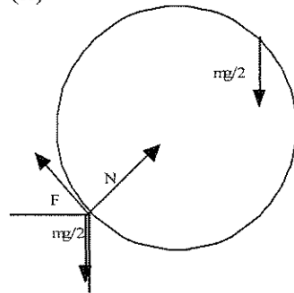
Resolving in the opposite direction to N , $mg\cos\theta - N = ma\dot{\theta}^2$,

and losing contact $N = 0$, so $a\dot{\theta}^2 = g\cos\theta$.

Thus from the energy equation $u^2 + 2ag = 3ag\cos\theta$ and so the hub has fallen

$$a - a\cos\theta = a - \frac{u^2 + 2ag}{3g} = \frac{ag - u^2}{3g} > 0, \text{ but is less than } a.$$

(ii)



As before $\frac{1}{2}\frac{m}{2}(2u)^2 + \frac{m}{2}g(2a) = \frac{1}{2}\frac{m}{2}(2a)^2\dot{\theta}^2 + \frac{m}{2}g(2a)\cos\theta$ and $0 = 2a\ddot{\theta} - g\sin\theta$,

and $mg\sin\theta - F = \frac{m}{2}(2a)\ddot{\theta}$ so $F = \frac{1}{2}mg\sin\theta$.

Also $mg\cos\theta - N = \frac{m}{2}(2a)\dot{\theta}^2$ and so when contact is lost $N = 0$, so $a\dot{\theta}^2 = g\cos\theta$,

$$u^2 + ag = 2ag\cos\theta,$$

and the hub has fallen $a - a\cos\theta = a - \frac{u^2 + ag}{2g} = \frac{ag - u^2}{2g} > 0$, but is less than a .



NextStepMaths.com

To view mark schemes, fully worked solutions and examiner's comments, and for more details about tutoring and other services offered, go to

NextStepMaths.com



So when $N = 0$, $\mu N = 0$, $F > 0$, but we require $F < \mu N$ not to slip, and hence slipping will certainly occur before it loses contact with the table.



NextStepMaths.com

To view mark schemes, fully worked solutions and examiner's comments, and for more details about tutoring and other services offered, go to [NextStepMaths.com](https://www.NextStepMaths.com)