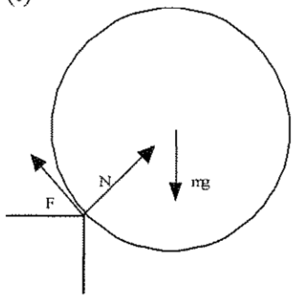
STEP III, 2007 Q11 MS

11. (i)



If the angle between mg and N is θ, then conserving energy and either differentiating the energy equation or taking moments about the point of contact yields

$\frac{1}{2}mu^2 + mga = \frac{1}{2}ma^2\dot{\theta}^2 + mga\cos\theta$ and $0 = a\ddot{\theta} - g\sin\theta$

Resolving in the opposite direction to F, $mg\sin\theta - F = ma\ddot{\theta}$ and so, from the second equation above, $F = 0$.

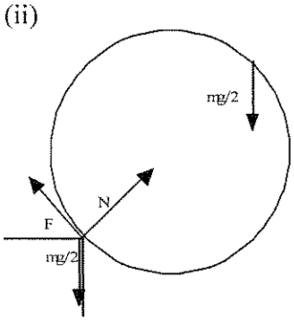
Resolving in the opposite direction to N, $mg\cos\theta - N = ma\dot{\theta}^2$,

and losing contact $N = 0$, so $a\dot{\theta}^2 = g\cos\theta$.

Thus from the energy equation $u^2 + 2ag = 3ag\cos\theta$ and so the hub has fallen

$a - a\cos\theta = a - \frac{u^2 + 2ag}{3g} = \frac{ag - u^2}{3g} > 0$, but is less than a.

(ii)



As before $\frac{1}{2}\frac{m}{2}(2u)^2 + \frac{m}{2}g(2a) = \frac{1}{2}\frac{m}{2}(2a)^2\dot{\theta}^2 + \frac{m}{2}g(2a)\cos\theta$ and $0 = 2a\ddot{\theta} - g\sin\theta$,

and $mg\sin\theta - F = \frac{m}{2}(2a)\ddot{\theta}$ so $F = \frac{1}{2}mg\sin\theta$.

Also $mg\cos\theta - N = \frac{m}{2}(2a)\dot{\theta}^2$ and so when contact is lost $N = 0$, so $a\dot{\theta}^2 = g\cos\theta$,

$u^2 + ag = 2ag\cos\theta$,

and the hub has fallen $a - a\cos\theta = a - \frac{u^2 + ag}{2g} = \frac{ag - u^2}{2g} > 0$, but is less than a.

So when $N = 0$, $\mu N = 0$, $F > 0$, but we require $F < \mu N$ not to slip, and hence slipping will certainly occur before it loses contact with the table.