

### STEP III, 2007 Q10 MS

10. Using uniform acceleration formulae with  $(\ddot{x}, \ddot{y}) = (-g \sin \phi, -g \cos \phi)$ , then

$$(x, y) = \left( Vt \cos \theta - \frac{1}{2}gt^2 \sin \phi, Vt \sin \theta - \frac{1}{2}gt^2 \cos \phi \right).$$

To return on the same path  $\dot{x} = 0$  when  $y = 0$ . So  $t = \frac{V \cos \theta}{g \sin \phi} = \frac{2V \sin \theta}{g \cos \phi}$

i.e.  $2 \tan \phi \tan \theta = 1$

Also using  $v^2 = u^2 + 2as$  in the  $x$  direction  $0 = V^2 \cos^2 \theta - 2gR \sin \phi$

i.e.  $R = \frac{V^2 \cos^2 \theta}{2g \sin \phi}$ .

Thus

$$\frac{2V^2}{gR} = 4 \sin \phi \sec^2 \theta = 4 \sin \phi (1 + \tan^2 \theta) = 4 \sin \phi \left( 1 + \frac{1}{4} \cot^2 \phi \right) = 4 \sin \phi \left( 1 + \frac{1}{4} (\operatorname{cosec}^2 \phi - 1) \right)$$

$$= 3 \sin \phi + \operatorname{cosec} \phi$$

Consider  $y = 3x + \frac{1}{x}, x > 0$ . By differentiation, this is least for  $x = \frac{1}{\sqrt{3}}$ .

Thus the least value of  $\frac{2V^2}{gR}$  is  $2\sqrt{3}$ , and the largest value of  $R$  is  $\frac{V^2}{\sqrt{3}g}$ .



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