

## STEP III, 2007, Q10

- 10 A particle is projected from a point on a plane that is inclined at an angle  $\phi$  to the horizontal. The position of the particle at time  $t$  after it is projected is  $(x, y)$ , where  $(0, 0)$  is the point of projection,  $x$  measures distance up the line of greatest slope and  $y$  measures perpendicular distance from the plane. Initially, the velocity of the particle is given by  $(\dot{x}, \dot{y}) = (V \cos \theta, V \sin \theta)$ , where  $V > 0$  and  $\phi + \theta < \pi/2$ . Write down expressions for  $x$  and  $y$ .

The particle bounces on the plane and returns along the same path to the point of projection. Show that

$$2 \tan \phi \tan \theta = 1$$

and that

$$R = \frac{V^2 \cos^2 \theta}{2g \sin \phi},$$

where  $R$  is the range along the plane.

Show further that

$$\frac{2V^2}{gR} = 3 \sin \phi + \operatorname{cosec} \phi$$

and deduce that the largest possible value of  $R$  is  $V^2/(\sqrt{3}g)$ .



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