Attempt this question independently, showing full and clear working.

STEP III 2007 Question 1

In this question, do not consider the special cases in which the denominators of any of your expressions are zero.

Express $\tan(\theta_1 + \theta_2 + \theta_3 + \theta_4)$ in terms of t_i, where $t_1 = \tan\theta_1$, etc.

Given that $\tan\theta_1$, $\tan\theta_2$, $\tan\theta_3$ and $\tan\theta_4$ are the four roots of the equation

$$at^4 + bt^3 + ct^2 + dt + e = 0$$

(where $a \neq 0$), find an expression in terms of a, b, c, d and e for $\tan(\theta_1 + \theta_2 + \theta_3 + \theta_4)$.

The four real numbers θ_1, θ_2, θ_3 and θ_4 lie in the range $0 \leqslant \theta_i < 2\pi$ and satisfy the equation

$$p\cos 2\theta + \cos(\theta - \alpha) + p = 0,$$

where p and α are independent of θ. Show that $\theta_1 + \theta_2 + \theta_3 + \theta_4 = n\pi$ for some integer n.