

STEP III, 2006, Q9 MS

9

Take O as the zero level for potential energy. Then PE of bead at B is mgy ; PE of particle at P is $mgr - mgl$.

For perpetual equilibrium, the PE must have the same value in any position, in particular its value at H; result follows.

Express equation shown in polar coordinates to get

$$r = \frac{2h}{1 + \sin \theta}$$

Differentiate and make $\dot{\theta}$ the subject so

$$\dot{\theta} = -\frac{\dot{r}(1 + \sin \theta)^2}{2h \cos \theta}.$$

These two expressions give the desired result.

By conservation of energy if PE is constant so is KE. Hence KE in a general position is equal to the initial value. That gives

$$V^2 = \left(r \dot{\theta} \right)^2 + 2\dot{r}^2$$

Speed of the particle at P is $\left| \dot{r} \right|$. Use the expressions for $V^2, \dot{\theta}$ to derive the required result.



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