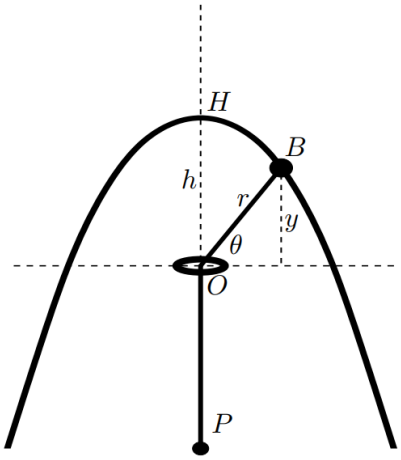
STEP III, 2006, Q9

9 A long, light, inextensible string passes through a small, smooth ring fixed at the point O. One end of the string is attached to a particle P of mass m which hangs freely below O. The other end is attached to a bead, B, also of mass m, which is threaded on a smooth rigid wire fixed in the same vertical plane as O. The distance OB is r, the distance OH is h and the height of the bead above the horizontal plane through O is y, as shown in the diagram.



The shape of the wire is such that the system can be in static equilibrium for all positions of the bead. By considering potential energy, show that the equation of the wire is $y + r = 2h$.

The bead is initially at H. It is then projected along the wire with initial speed V. Show that, in the subsequent motion,

$$\dot{\theta} = -\frac{h\dot{r}}{r\sqrt{rh - h^2}}$$

where θ is given by $\theta = \arcsin(y/r)$.

Hence show that the speed of the particle P is $V\left(\frac{r-h}{2r-h}\right)^{\frac{1}{2}}$.

[*Note that* $\arcsin\theta$ *is another notation for* $\sin^{-1}\theta$.]