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8	Use (iv) with $f(x)\equiv 1, g(x)\equiv 1$ to show that $\Delta 1 = 0$. Use (iii) with $\lambda \equiv k, f(x)\equiv 1$ to show that $\Delta k = 0$. By (iv), (i) $\Delta x^2 = 2x$; ditto $\Delta x^3 = 3x^2$. Now show $\Delta kx^n = knx^{n-1}$ by induction. Initial step is $\Delta k = 0$; inductive hypothesis is that $\Delta kx^N = kNx^{N-1}$. Use (iii) and (iv) with hypothesis to show that $\Delta kx^{N+1} = k(N+1)x^N$. Now express any $P_k(x)$, a polynomial of degree k, as a sum of such powers, and so use (ii) to establish required result.